Calibrated sensitivity models

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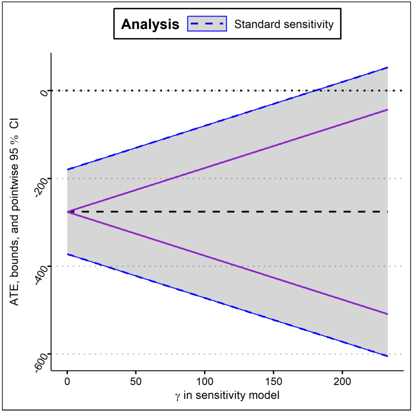
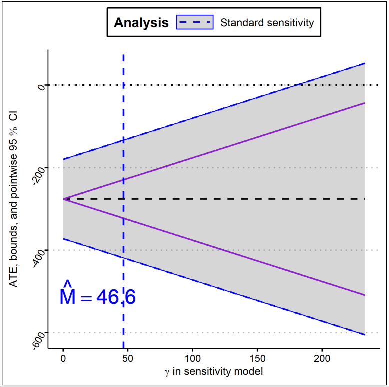
Partial identification sensitivity model: $U \leq \gamma$

E.g., U is odds ratio of propensity score [Rosenbaum, 2002, Tan, 2006]



Issue #1: With nonparametric methods, difficult to interpret γ (e.g., how big is $\gamma = 190$?)

Post hoc calibration: estimate measured confounding M



Issue #2: Uncertainty in $\widehat{M}$ is unaccounted for

Solution: calibrated sensitivity models

Sensitivity model: $U \leq \gamma$

e.g., U is odds ratio of propensity score [Rosenbaum, 2002, Tan, 2006]

Calibrated sensitivity models: $\boxed{U \leq \Gamma M}$
$\rightarrow$ put measured confounding in the model!

Issue #1: γ difficult to interpret

Γ is interpretable bound on **unit-less** ratio U/M

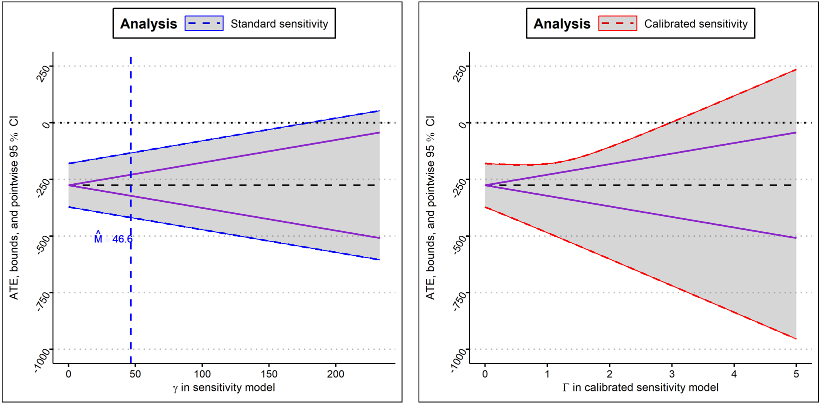
Issue #2: uncertainty in $\widehat{M}$ is unaccounted for

One can develop methods to account for uncertainty in estimating measured confounding

Issue #3: choice of measured confounding not justified

Clearer researchers must justify choice of measured confounding because explicit assumption in model

Accounting for uncertainty in $\widehat{M}$ can change results!



$\rightarrow$ red wider than blue $\Longrightarrow$ less robust to unmeasured confounding

$\rightarrow$ red narrower than blue $\Longrightarrow$ more robust to unmeasured confounding

Shape of red CI depends on covariance between estimators for bounds and measured confounding M

Setup

Setup: data and assumptions

Observe $Z_i = \{X_i, A_i, Y_i\}$ for $i = 1, ..., n$ where $Z_i \overset{iid}{\sim} \mathcal{P}$

$X \in \mathbb{R}^d$ are d-**dimensional covariates**
$A \in \{0, 1\}$ is a **binary treatment**
$Y \in \mathbb{R}$ is an **outcome**
Y^a is the **potential outcome** under treatment a
W are **unmeasured confounders**

Nuisance functions:
$\pi(X) = \mathbb{P}(A = 1 \mid X)$ is the **propensity score**
$\mu_a(X) = \mathbb{E}(Y \mid A = a, X)$ is the **outcome regression function**

Causal assumptions:

1. **Consistency:** $Y = Y^a$ if $A = a$
2. **Positivity:** $\exists\ \varepsilon > 0$ s.t. $\mathbb{P}\{\varepsilon \leq \pi(X) \leq 1 - \varepsilon\} = 1$.

Average Treatment Effect (ATE): $\psi_* = \mathbb{E}\left(Y^1 - Y^0\right)$

Calibrated sensitivity models

Model choices

Calibrated sensitivity models: $\boxed{U \leq \Gamma M}$

where

- U is unmeasured confounding,
- Γ is the sensitivity parameter
- M is measured confounding (analogous to U)

Example: [Rosenbaum, 2002]

$$\underbrace{\sup_{x,w,\widetilde{w}} \left| \log \left[\frac{\text{odds}\{\pi(x,w)\}}{\text{odds}\{\pi(x,\widetilde{w})\}} \right] \right|}_{U} \leq$$

$$\Gamma \underbrace{\left(\max_{j \in \{1,\dots,d\}} \sup_{x_{-j},x_j,\widetilde{x}_j} \left| \log \left[\frac{\text{odds}\{\pi(x_{-j},x_j)\}}{\text{odds}\{\pi(x_{-j},\widetilde{x}_j)\}} \right] \right| \right)}_{M}$$

x_j is j^{th} covariate and x_{-j} is $d-1$ covariates with j^{th} removed

Model choices: measured confounding

Measured confounding M**:**

- Which subsets measured confounding includes,
- Whether measured confounding is a max or an avg

$$\text{Maximum leave-one-out} = \max_{j \in \{1,\ldots,d\}} \sup_{x_{-j}, x_j, \widetilde{x}_j} \left| \log \left[\frac{\text{odds}\{\pi(x_{-j}, x_j)\}}{\text{odds}\{\pi(x_{-j}, \widetilde{x}_j)\}} \right] \right|$$

$$\text{Average leave-some-out} = \frac{1}{|\mathcal{S}|} \sum_{S \in \mathcal{S}} \sup_{x_{-S}, x_S, \widetilde{x}_S} \left| \log \left[\frac{\text{odds}\{\pi(x_{-S}, x_S)\}}{\text{odds}\{\pi(x_{-S}, \widetilde{x}_S)\}} \right] \right|$$

→ exclude **multiple covariates** because suspect **correlated** and **joint effect** better proxy for unmeasured confounder

S is an index on $\{1, \ldots, d\}$, x_S are covariates corresponding to S (e.g., $x_{\{1,2\}}$ are first two covs), and x_{-S} are $d - |S|$ covs with x_S removed.

Partial Identification

(Proposition, informal) Bounds on the ATE

$\psi_* \in [\mathcal{L}(\Gamma), \mathcal{U}(\Gamma)]$

$$\mathcal{U}(\Gamma) = \mathbb{E}\Big[(1-A)\underbrace{\theta_1^+\{X; \exp(\Gamma M)\}}_{\text{depends on } M!} - A\theta_0^-\{X; \exp(\Gamma M)\}\Big]$$

$$\mathcal{L}(\Gamma) = \mathbb{E}\Big[(1-A)\theta_1^-\{X; \exp(\Gamma M)\} - A\theta_0^+\{X; \exp(\Gamma M)\}\Big]$$

where, e.g., $\theta_1^+(X; t)$ **is the upper bound on** $\mathbb{E}(Y^1 \mid A = 0, X)$ with parameter t [Yadlowsky et al., 2022]

Estimation and inference

Establishing convergence guarantees

1. **Establish that the bounds $\mathcal{U}(\Gamma)$ and $\mathcal{L}(\Gamma)$ are differentiable with respect to M**

$\rightarrow$ can use Taylor's theorem (/delta method) when providing error guarantees

$\rightarrow$ involved proof when nuisance functions **depend on** M, like $\theta\{X; \exp(\Gamma M)\}$

2. **Establish estimator for M is regular and asymptotically linear** (RAL) under doubly-robust-style conditions

$\rightarrow$ Use efficient influence functions

$\implies$ bias is product of nuisance function errors (e.g., $\|\widehat{\pi} - \pi\|\|\widehat{\mu} - \mu\|$)

3. **Establish estimator for bound $\mathcal{U}(\Gamma)$ is RAL** under doubly-robust-style conditions

$\rightarrow$ Use steps #1 and #2 and efficient influence functions

(Theorem, informal) Convergence guarantees for estimators for bounds, and inference for the ATE

Under doubly-robust-style conditions on the nuisance function estimators, (e.g., $\|\widehat{\pi} - \pi\| \|\widehat{\mu} - \mu\| = o_{\mathbb{P}}(n^{-1/2})$),

$$\widehat{\mathcal{U}}(\Gamma) - \mathcal{U}(\Gamma) = (\mathbb{P}_n - \mathbb{P})\varphi_{\mathcal{U}}(Z) + o_{\mathbb{P}}(n^{-1/2})$$
$$\widehat{\mathcal{L}}(\Gamma) - \mathcal{L}(\Gamma) = (\mathbb{P}_n - \mathbb{P})\varphi_{\mathcal{L}}(Z) + o_{\mathbb{P}}(n^{-1/2})$$

$\rightarrow$ $\varphi_{\mathcal{U}}$ and $\varphi_{\mathcal{L}}$ account for uncertainty in estimating M

Constructing confidence intervals for ATE:
$\rightarrow$ Intersection one-sided Wald-type intervals for $\mathcal{U}$ and $\mathcal{L}$.

Summary

- To solve issues with standard sensitivity analyses and post hoc calibration, we proposed novel **calibrated sensitivity models**: $\boxed{U \leq \Gamma M}$,
- Discussed model choices within a calibrated framework, in particular for measured confounding,
- Partially identified the ATE,
- Developed methods for estimation and inference which account for uncertainty in $\widehat{M}$

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Thank you for your attention!

References

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